

Mathematics

Education must equip all students with mathematical skills that provide them with the flexibility, adaptability, and creativity to function as productive citizens in the changing technological society of the twenty-first century. Mathematical skills must extend beyond the ability to calculate into the use of mathematics to investigate, analyze, and interpret.

Thinking mathematically is critical to every life skill from balancing a checkbook to understanding the newspaper. People use math skills daily to identify problems, look for information that will help solve the problems, consider a variety of solutions, and communicate the best solution to others.

A math classroom should provide practical experience in mathematical skills that are a bridge to the real world, as well as explorations which develop an appreciation of the beauty and value of mathematics. Using a variety of tools, such as calculators, computers, and hands-on materials, under the guidance of a skillful teacher creates a rich mathematical learning environment. Such an environment will help to prepare students for a world where using calculators and computers to carry out mathematical procedures is commonplace, a world where mathematics is rapidly growing and extensively being applied in diverse fields.

Maine should expect its students to enjoy, appreciate, and use mathematics, just as it expects them to enjoy, appreciate, and use music, art, and literature. Students who are challenged to reach these goals will be better prepared for a future in which mathematics will be increasingly important in all areas of endeavor.

A. NUMBERS AND NUMBER SENSE

Students will understand and demonstrate a sense of what numbers mean and how they are used. *Numbers are used to describe and interpret phenomena. Building a sense of number relationships is essential for developing the ability to deal with any set of numbers. Number sense involves understanding the meaning of numbers, relationships among numbers, relative number magnitudes, and the effects of operations on numbers. Skilled estimation is also an important component of number sense.*

B. COMPUTATION

Students will understand and demonstrate computation skills. *Understanding the fundamental operations of addition, subtraction, multiplication, and division is central to knowing mathematics. Proficiency in computational skills is essential to problem-solving and other mathematical activities. Estimating, evaluating reasonableness of answers, and obtaining accuracy in calculations are included in this proficiency. Understanding relationships in operations allows students greater facility with mental computation. Computational skill promotes efficient and confident learners.*

C. DATA ANALYSIS AND STATISTICS

Students will understand and apply concepts of data analysis. *We are faced with massive quantities of information which must be selected, sorted, and analyzed to reach conclusions. Sound decision making requires the ability to collect data effectively, organize data, discover patterns, summarize trends, make inferences, draw conclusions, and make predictions. The ethical use of statistics is a paramount concern in the Information Age.*

D. PROBABILITY

Students will understand and apply concepts of probability. *Probability is the study of uncertainty. Informed consumers of information understand the basic principles of probability. People need to understand the uncertainties and limitations involved when drawing conclusions from data.*

E. GEOMETRY

Students will understand and apply concepts from geometry. *Geometry is the study of the spatial world and its symmetries. The ideas of geometry are used to describe, interpret, represent, and change the spatial world in which we live. The understanding and development of spatial and visual skills strengthens problem-solving abilities.*

Mathematics

F. MEASUREMENT

Students will understand and demonstrate measurement skills. *Measurement is valuable as an integrating skill throughout the curriculum and in everyday life. The use of estimation is vital in determining the reasonableness of measurement. Measurement attributes (e.g., length, volume, minutes), units, and tools enhance the ability to describe and understand the world.*

G. PATTERNS, RELATIONS, FUNCTIONS

Students will understand that mathematics is the science of patterns, relationships, and functions. *Relationships are central to mathematical understanding. A study of patterns often reveals regularity, indicating the presence of a mathematical relationship. Studying relationships allows students to make generalizations and predictions about phenomena and occurrences.*

H. ALGEBRA CONCEPTS

Students will understand and apply algebraic concepts. *Algebra and analytic thinking are fundamental tools for working in and thinking about mathematics. These tools provide ways to generalize and predict problem solutions when not all information is known. Taught within the context of mathematical and practical applications, the concept of functions is a unifying theme for algebraic concepts.*

I. DISCRETE MATHEMATICS

Students will understand and apply concepts in discrete mathematics. *Discrete mathematics studies discrete processes (e.g., all possible bus routes in a school district). This study includes the exploration of diagrams, networks, and flowcharts that students construct to model situations or use for planning, scheduling, and decision making. Three main concerns of discrete mathematics are: existence (Is there a solution?), counting (How many solutions are there?), and efficiency (What is the best solution?).*

J. MATHEMATICAL REASONING

Students will understand and apply concepts of mathematical reasoning. *Reasoning is fundamental to the knowing and doing of mathematics. To give more students access to mathematics as a powerful way of making sense of the world, it is essential that an emphasis on reasoning pervade all mathematics. Students need a great deal of time and many experiences to develop their ability to construct valid arguments in problem settings and to evaluate the arguments of others.*

K. MATHEMATICAL COMMUNICATION

Students will reflect upon and clarify their understanding of mathematical ideas and relationships. *Communication plays a key role in helping make important connections among physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas. Providing individual and collaborative opportunities for discussions about issues, people, and the cultural implications of mathematics reinforce student understanding of the connection between mathematics and our society.*

Mathematics

A. NUMBERS AND NUMBER SENSE

Students will understand and demonstrate a sense of what numbers mean and how they are used. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Demonstrate an understanding of what numbers mean (e.g., that the number 7 stands for a group of objects).
2. Understand the many uses of numbers (e.g., prices, recipes, measurement, directions in play).
3. Order, compare, read, group, and apply place value concepts to numbers up to 1,000.
4. Determine reasonableness of results when working with quantities.

EXAMPLES

- Show that 6 is larger than 3, using beans in a cup.
- Explain different ways to make 263, using hundreds, tens, and ones.

ELEMENTARY GRADES 3-4

1. Read, compare, order, classify, and explain whole numbers up to one million.
2. Read, compare, order, classify, and explain simple fractions through tenths.
3. Demonstrate knowledge of the meaning of decimals and integers and an understanding of how they may be used.

EXAMPLES

- Using pattern blocks, represent equivalent fractions, such as $\frac{1}{3}$, $\frac{2}{6}$, $\frac{4}{12}$
- Show how three pizzas can be shared equally by four people.

MIDDLE GRADES 5-8

1. Use numbers in a variety of equivalent and interchangeable forms (e.g., integer, fraction, decimal, percent, exponential, and scientific notation) in problem-solving.
2. Demonstrate understanding of the relationships among the basic arithmetic operations on different types of numbers.
3. Apply concepts of ratios, proportions, percents, and number theory (e.g., primes, factors, and multiples) in practical and other mathematical situations.
4. Represent numerical relationships in graphs, tables, and charts.

EXAMPLES

- Use integers to write a play-by-play description of a game (e.g., football, soccer, or golf).
- Respond to the following in a journal: How can you get a smaller answer when you multiply? How can you get a larger answer when you divide?
- Estimate a 15% tip for a meal costing \$38.60.

SECONDARY GRADES

1. Describe the structure of the real number system and identify its appropriate applications and limitations.
2. Explain what complex numbers (real and imaginary) mean and describe some of their many uses.

EXAMPLE

- Given two numbers such as $\frac{1}{2}$ and $\frac{1}{3}$, describe the real numbers between them.

Mathematics

B. COMPUTATION

Students will understand and demonstrate computation skills. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Use and apply estimation with quantities, measurements, computations, and problem-solving.
2. Use multiple strategies in solving problems involving addition and subtraction of whole numbers.
3. Show understanding of addition and subtraction by using a variety of materials, strategies, and symbols.

EXAMPLES

- Estimate reasonably and count accurately the number of seeds in a container.
- Solve a problem such as: we have 24 seats and 19 children. How many seats will be empty after all the children are in their chairs?
- Show different coin combinations to make 75¢.

ELEMENTARY GRADES 3-4

1. Solve multi-step, real-life problems using the four operations with whole numbers.
2. Solve real-life problems involving addition and subtraction of simple fractions.
3. Demonstrate and explain the problem-solving process using appropriate tools and technology and defend the reasonableness of results.
4. Develop proficiency with the facts and algorithms of the four operations on whole numbers using mental math and a variety of materials, strategies, and technologies.

EXAMPLES

- Solve problems such as finding the number of rectangular arrangements for 36 like objects.
- Organize a budget for a project.

MIDDLE GRADES 5-8

1. Compute and model all four operations with whole numbers, fractions, decimals, sets of numbers, and percents, applying the proper order of operations.
2. Create, solve, and justify the solution for multi-step, real-life problems including those with ratio and proportion.

EXAMPLES

- Show that there must have been at least one misprint in a newspaper report on an election that reads:
 - Yes votes 13,657 (42%)
 - No votes 186,491 (58%)and suggest two specific places a misprint may have occurred.
- On a number line, name the point located midway between $\frac{1}{4}$ and 6.

SECONDARY GRADES

1. Use various techniques to approximate solutions, determine the reasonableness of answers, and justify the results.
2. Explain operations with number systems other than base ten.

EXAMPLE

- If 10% of U.S. citizens have a certain trait, and four out of five with the trait are men, determine what proportion of men have the trait and what proportion of women have the trait. Explain whether the answer depends on the proportion of U.S. citizens who are women, and if so, how?

Mathematics

C. DATA ANALYSIS AND STATISTICS

Students will understand and apply concepts of data analysis. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Formulate and solve problems by collecting, arranging, and interpreting data.
2. Make tallies and graphs of information gathered from immediate surroundings.

EXAMPLE

- Make a graph to represent the number of family members for students in the class and use this graph to determine the amount of refreshments needed for a family-night presentation.

ELEMENTARY GRADES 3-4

1. Make generalizations and draw conclusions using various types of graphs, charts, and tables.
2. Read and interpret displays of data.

EXAMPLE

- Predict the number of buttons per student in the room. Collect data on the number of buttons for each student in the room, display the data on a line plot, and analyze it to determine the average number of buttons per student.

MIDDLE GRADES 5-8

1. Organize and analyze data using mean, median, mode, and range.
2. Assemble data and use matrices to formulate and solve problems.
3. Construct inferences and convincing arguments based on data.

EXAMPLES

- Conduct an experiment to determine the effects of fertilizer on plant growth, recording and analyzing information on charts and graphs.
- Using the height of students in the room, calculate the mean, median, mode, and range.

SECONDARY GRADES

1. Determine and evaluate the effect of variables on the results of data collection.
2. Predict and draw conclusions from charts, tables, and graphs that summarize data from practical situations.
3. Demonstrate an understanding of concepts of standard deviation and correlation and how they relate to data analysis.
4. Demonstrate an understanding of the idea of random sampling and recognition of its role in statistical claims and designs for data collection.
5. Revise studies to improve their validity (e.g., in terms of better sampling, better controls, or better data analysis techniques).

EXAMPLES

- Draw a scatterplot of the height of each student in the class vs. their shoe length and find the line of best fit using a graphics calculator or computer software.
- Design and conduct an experiment to estimate the population of clams in a given clam flat.

Mathematics

D. PROBABILITY

Students will understand and apply concepts of probability. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Use concepts of chance and record outcomes of simple events.

EXAMPLE

- Investigate the possible and likely outcomes when rolling a number cube.

ELEMENTARY GRADES 3-4

1. Explain the concept of chance in predicting outcomes.
2. Estimate probability from a sample of observed outcomes and simulations.

EXAMPLE

- Determine the combinations that could occur when four coins are tossed, and predict which of the combinations would occur most often. Conduct an experiment to show how close the predictions are to actual events.

MIDDLE GRADES 5-8

1. Find the probability of simple events and make predictions by applying the theories of probability.
2. Explain the idea that probability can be represented as a fraction between and including zero and one.
3. Use simulations to estimate probabilities.
4. Find all possible combinations and arrangements involving a limited number of variables.

EXAMPLES

- Develop and analyze games of chance for a school carnival.
- Determine how many license plates are possible if the first two symbols are letters and the last four are numbers.

SECONDARY GRADES

1. Find the probability of compound events and make predictions by applying probability theory.
2. Create and interpret probability distributions.

EXAMPLE

- Determine the probability that a 90% free throw shooter will make exactly one of his/her upcoming two free throws.

Mathematics

E. GEOMETRY

Students will understand and apply concepts from geometry. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Describe, model, and classify 2D shapes and selected 3D figures.
2. Investigate and predict the results of combining, dividing, and changing 2D shapes.
3. Use positional words to describe the relationship of two or more objects (e.g., over, under, beside, to the left).

EXAMPLES

- Find all the shapes you can make with five squares of the same size if the sides touch completely.
- Create symmetrical designs with pattern blocks and indicate lines of symmetry.

ELEMENTARY GRADES 3-4

1. Describe, model, and classify shapes and figures using applicable properties.
2. Experiment with shapes and figures to make generalizations regarding congruency, symmetry, and similarity.
3. Use transformations such as slides, flips, and rotations.
4. Use the properties of shapes and figures to describe the physical world.

EXAMPLES

- Design a nine patch quilt in which each patch is a nine inch square. Squares, triangles, rectangles, and/or parallelograms can be used for the design of each square.
- Create symmetrical designs with pattern blocks and indicate the lines of symmetry.

MIDDLE GRADES 5-8

1. Compare, classify, and draw two dimensional shapes and three dimensional figures.
2. Apply geometric properties to represent and solve real-life problems involving regular and irregular shapes.
3. Use a coordinate system to define and locate position.
4. Use the appropriate geometric tools and measurements to draw and construct two and three dimensional figures.

EXAMPLES

- Collect magazine pictures of different styles of architecture and identify all the geometric figures and relationships seen in each building.
- Display data with an accurately drawn and divided pie chart.

SECONDARY GRADES

1. Draw coordinate representations of geometric figures and their transformations.
2. Use inductive and deductive reasoning to explore and determine the properties of and relationships among geometric figures.
3. Apply trigonometry to problem situations involving triangles and periodic phenomena.

Mathematics

F. MEASUREMENT

Students will understand and demonstrate measurement skills. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Estimate and measure length, time, temperature, weight, and capacity.
2. Identify and give the value of different coins.
3. Select standard and non-standard tools for determining length, time, temperature, weight, and capacity, and use them to solve every day problems.

EXAMPLE

- Put five objects, such as books, rocks, or pumpkins, in rank order by weight, first by estimating and then by measuring exactly.

ELEMENTARY GRADES 3-4

1. Solve and justify solutions to real-life problems involving the measurement of time, length, area, perimeter, weight, temperature, mass, capacity, and volume.
2. Select measuring tools and units of measurement that are appropriate for what is being measured.

EXAMPLES

- Find all of the rectangular areas, measured in whole square inches, for a particular perimeter, such as 24 inches, and see if the same pattern of shapes holds for another perimeter.
- Determine what time it is necessary to leave the house for the fifteen minute walk to the bus stop and 1-1/4 hour bus ride to the 7:30 concert. Explain the answer.

MIDDLE GRADES 5-8

1. Demonstrate the structure and use of systems of measurement.
2. Develop and use concepts that can be measured directly, or indirectly (e.g., the concept of rate).
3. Demonstrate an understanding of length, area, volume, and the corresponding units, square units, and cubic units of measure.

EXAMPLES

- Calculate the rate of speed of a moving object after measuring the distance traveled and the elapsed time.
- Examine areas that can be enclosed using 24 feet of fencing and figuring out the maximum area.
- Calculate the volume and surface areas of cones and pyramids.

SECONDARY GRADES

1. Use measurement tools and units appropriately and recognize limitations in the precision of the measurement tools.
2. Derive and use formulas for area, surface area, and volume of many types of figures.

EXAMPLES

- Discover and explore the distance formula using the Pythagorean Theorem.
- Using generalizations, compare the formula for the area of an n-sided, regular polygon to the formula for the area of a circle.

Mathematics

G. PATTERNS, RELATIONS, FUNCTIONS

Students will understand that mathematics is the science of patterns, relationships, and functions. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Recognize, describe, extend, copy, and create a wide variety of patterns.
2. Explore the use of variables and open sentences to describe relationships.
3. Represent and describe both geometric and numeric relationships.

EXAMPLE

- Recognize and describe the inverse relationship between addition and subtraction, e.g. $3 + __ = 7$, $__ + 3 = 7$, $7 - 3 = __$, and $7 - __ = 3$.
- Show how the letters “aab, aab, ...,” can represent the pattern “metal, metal, plastic, ...,” “leaf, leaf, rock, ...,” or many other patterns.

ELEMENTARY GRADES 3-4

1. Use the patterns of numbers, geometry, and a variety of graphs to solve a problem.
2. Use variables and open sentences to express relationships.

EXAMPLES

- Find, make, and describe linear patterns on the 99-chart, for example 4, 14, 24, 34.

MIDDLE GRADES 5-8

1. Describe and represent relationships with tables, graphs, and equations.
2. Analyze relationships to explain how a change in one quantity can result in a change in another.
3. Use patterns and multiple representations to solve problems.

EXAMPLES

- Collect data on the cost of first class postage stamps for a one hundred year period of time and predict future costs for such stamps.
- Determine the units digit (ones' place) of $(3)^{78}$.

SECONDARY GRADES

1. Create a graph to represent a real-life situation and draw inferences from it.
2. Translate and solve a real-life problem using symbolic language.
3. Model phenomena using a variety of functions (linear, quadratic, exponential, trigonometric, etc.).
4. Identify a variety of situations explained by the same type of function.

EXAMPLES

- Express the diameter of a circle as a function of its area and sketch a graph.
- Determine which of two ways of rolling a 8.5”x11” piece of paper into a cylinder gives the greater volume and whether there is a way to get even greater volume using a sheet of paper with the same area but different shape.

Mathematics

H. ALGEBRA CONCEPTS

Students will understand and apply algebraic concepts. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Make drawings for problem situations and mathematical expressions in which there is an unknown, using a variety of tools and approaches.
2. Use language and symbols to express numerical and other relationships.

EXAMPLE

- Show all the ways to make 10 (e.g., $2 + x = 10$, $3 + x = 10$, and so forth) by using blocks or other objects to demonstrate the mathematical statements.

ELEMENTARY GRADES 3-4

1. Develop and evaluate simple formulas in problem-solving contexts.
2. Find replacements for variables that make simple number sentences true.

EXAMPLE

- Plot points on a coordinate graph according to the convention that (x,y) refers to the intersection of a given vertical line and a given horizontal line.

MIDDLE GRADES 5-8

1. Use the concepts of variables and expressions.
2. Solve linear equations using concrete, informal, and formal methods which apply the order of operations.
3. Analyze tables and graphs to identify properties and relationships in a practical context.
4. Use graphs to represent two-variable equations.
5. Demonstrate an understanding of inequalities and non-linear equations.
6. Find solutions for unknown quantities in linear equations and in simple equations and inequalities.

EXAMPLES

- Study the steepness of wheelchair ramps and stairs.
- Solve for x : $3x - 5 = 23 - x$.

SECONDARY GRADES

1. Use tables, graphs, and spreadsheets to interpret expressions, equations, and inequalities.
2. Investigate concepts of variation by using equations, graphs, and data collection.
3. Formulate and solve equations and inequalities.
4. Analyze and explain situations using symbolic representations.

EXAMPLES

- Use measurements from shopping carts which are nested together to find a formula for the number of carts that will fit in a given space and a formula for the amount of space needed for a given number of carts.
- Solve the following problem: Given the formula for height of an object thrown upward with velocity v : $h = h_0 + vt + (1/2)gt^2$, use quadratic functions and the quadratic formula to answer questions about the motion of projectiles and falling objects.

Mathematics

I. DISCRETE MATHEMATICS

Students will understand and apply concepts in discrete mathematics. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Classify sets of objects into two or more groups using their attributes.
2. Create and use an organized list to determine possible outcomes or solve problems.

EXAMPLES

- Sort sets of tiles by color, size, and shape.
- Determine the possible arrangements for a triple ice cream cone given three flavors of ice cream.

ELEMENTARY GRADES 3-4

1. Create and use organized lists, tree diagrams, Venn diagrams, and networks.
2. Give examples of infinite and finite solutions.

EXAMPLE

- Use a Venn diagram to record the correlation between students who wore a sweater to school and students who walked to school.

MIDDLE GRADES 5-8

1. Create and use networks to explain practical situations or solve problems.
2. Identify patterns in the world and express these patterns with rules.

EXAMPLE

- Use graphs and matrices to determine delivery routes from Augusta to other major cities in Maine with a combination of one way and round-trip routes.

SECONDARY GRADES

1. Use linear programming to find optimal solutions to a system.
2. Use networks to find solutions to problems.
3. Apply strategies from game theory to problem-solving situations.
4. Use matrices as tools to interpret and solve problems.

EXAMPLE

- Given a decreasing linear relationship between the selling price of a magazine and the number of people who will buy it, and given a fixed cost per copy that goes to production, analyze the profitability of the product and recommend a price range.

Mathematics

J. MATHEMATICAL REASONING

Students will understand and apply concepts of mathematical reasoning. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Describe a simple argument's strengths and weaknesses.
2. Distinguish between "important" and "unimportant" mathematical information.

EXAMPLE

- Solve a simple word problem by finding and using the relevant information (Jack has 3 red marbles, 2 green marbles, and 4 toy cars. How many marbles does he have? $3+2=5$.)

ELEMENTARY GRADES 3-4

1. Demonstrate an understanding that support for a claim should be based on evidence of various types (e.g., from logical processes, from measurement, or from observation and experimentation).

EXAMPLE

- Using data from the classroom and the entire school, prepare a report, including graphs, charts, and diagrams, on the optimal number and location of recycling containers.

MIDDLE GRADES 5-8

1. Support reasoning by using models, known facts, properties, and relationships.
2. Demonstrate that multiple paths to a conclusion may exist.

EXAMPLE

- Prepare proposals for a fixed-height bridge and a draw bridge. Make recommendations after considering total cost, steepness of incline, traffic patterns, time of construction, etc.

SECONDARY GRADES

1. Analyze situations where more than one logical conclusion can be drawn from data presented.

EXAMPLE

- Given information about travel patterns in a local community, develop a convincing proposal for the logical placement of a bypass.

Mathematics

K. MATHEMATICAL COMMUNICATION

Students will reflect upon and clarify their understanding of mathematical ideas and relationships. Students will be able to:

ELEMENTARY GRADES Pre-K-2

1. Use numerals and symbols ($>$, $<$, $=$, $+$, $-$) to report numerical data and relationships.

ELEMENTARY GRADES 3-4

1. Use simple tables and graphs to communicate ideas and information in presentations in a concise and clear manner.

MIDDLE GRADES 5-8

1. Translate relationships into algebraic notation.
2. Use statistics, tables, and graphs to communicate ideas and information in convincing presentations and analyze presentations of others for bias or deceptive presentation.

SECONDARY GRADES

1. Restate, create, and use definitions in mathematics to express understanding, classify figures, and determine the truth of a proposition or argument.
2. Read mathematical presentations of topics within the Learning Results with understanding.

EXAMPLES:

- Having read the definition of “kite”, a student analyzes a collection of figures to decide which are kites. The student then proceeds to apply the kite definition to the families of quadrilaterals to determine which are kites and why.
- Student reads a manual or math text to successfully learn a new procedure.